Course of Advanced Automation and Control

Exam for the students of the a.y. 2016/2017

July 05, 2017

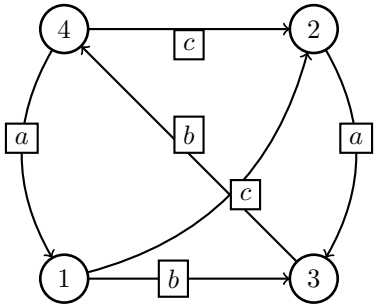
Surname _____ Name _____

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Mr. Grecchi is planning his 10 days (9 nights) of vacation to the seaside in Tuscany. He has a maximum budget of 1000€. He found two types of accommodations: an Airbnb and a 3 star hotel. The Airbnb costs 70€/night but does not include breakfast nor a private parking. The price of a breakfast in a local bar is around 6€/day. The price of a city parking is 10€/day. The hotel costs 80€/night and includes both the parking and the breakfast. Mr. Grecchi can go to the seaside either by train or by car (in this second case, if he chooses the Airbnb, he will need to pay for the city parking). The cost of the round trip by train is 65€. In case he goes by train, he will take a taxi from the station to his accommodation. The cost of the taxi is 40€ for the round trip. In case he decides to go by car, the cost is of 50€ for the gasoline plus 46€ for the highway. The objective of Mr. Grecchi is to minimize his expenses. Please formulate the problem above as a mixed integer linear program to support the decision-making process of Mr. Grecchi.

   Very important note: while formulating the problem above, you will obtain bilinear terms like $\delta_a\delta_b$ with δ_a and δ_b binary variables indicating the choice of accommodation and transportation. In order the problem to be an MILP, such term needs to disappear from the problem and be replaced by a new variable δ_c subject to inequality constraints which will relate δ_c to δ_a and δ_b. Please include these constraints (which we have seen during the course) in your final formulation.

2. Consider the automaton in the figure above.



   where $C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values. Let the intermediate cost $g(x, u)$ and the terminal cost $g_3(x)$ be given by

$g(x,u)$	a	b	c
1	-	3	2
2	3	-	-
3	-	-	4
4	1	-	2

$$g_3(x) = \begin{cases} 4 & \text{if } x = 1 \\ 3 & \text{if } x = 2 \\ 1 & \text{if } x = 3 \\ 2 & \text{if } x = 4 \end{cases}$$

   2.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0,u_1,u_2} g_3(x_3) + \sum_{k=0}^{2} g(x_k, u_k)$$

   using dynamic programming.

2.2 Compute an optimal control sequence for $x_0 = 3$ and compute the optimal cost value.

3. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{lll} \max_{x_1,\delta_1,\delta_2} & x_1 + 2\delta_1 + 4\delta_2 & \\ & \delta_2 + \delta_1 & \leq 1.5 \\ & \delta_1, \delta_2 \in \{0,1\} & \\ & x_1 \geq 0 & \end{array}$$